Continuous variable cloning via network of parametric gates

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We propose an experimental scheme for the cloning machine of continuous quantum variables through a network of parametric amplifiers working as input-output four-port gates.

Since the seminal paper of Bužek and Hillery [1], much theoretical work has been done about quantum cloning [2–6]. In Ref. [5] the problem of copying the state of a system with continuous variables has been studied, and a unitary transformation that clones coherent states with the same fidelity (equal to 2/3) has been found. However, proposals of its experimental realization have not appeared yet. In this Letter, we propose an experimental scheme to implement such a new kind of cloning. We show that a network of three parametric amplifiers, each of them working as an input-output four-port "gate", under suitable gain conditions realizes the one-to-two cloning machine for "distinguishable" clones. In fact, optimal cloning machines can be achieved using parametric down-converters, in two different ways. The first way, proposed in Ref. [7], uses a single "quantum-injected" parametric amplifier in a configuration already used in many experiments [8], where a one-photon state is down-converted into a many-photons entangled state. This contains clones of the injected state, which are supported by indistinguishable photons [9]. This way can then be used for measurements of permutation-invariant observables on clones, and allows one to study the clones statistics [10]. The second way is the subject of this Letter: a one-to-two cloning machine for distinguishable clones, based on parametric gates.

A relevant application of universally covariant cloning is eavesdropping for quantum cryptography [2]. Moreover, quantum cloning is of practical interest as a tool to engineer novel scheme for joint measurements. However, universal covariant cloning is not ideal for such purpose, and a suitable non universal cloning is needed [11]. If one wants to use quantum cloning to realize joint measurements, cloning must be optimized for a reduced covariance group, depending on the desired joint measurement, such that measurements on cloned copies are equivalent to optimal joint measurements on the original. As we will show, this is the case of the cloning map proposed in Ref. [5], which is covariant only with respect to the Weyl-Heisenberg group represented by the displacement operator, and which is optimal for the joint measurement of conjugated quadratures. Hence, measures of quality other than fidelity should be used for optimization, depending on the final use of the output copies. This is also indicated by recent studies of copying machines designed for information transfer [12]. Notice also that the major problem in quantum teleportation—the Bell measurement—may need a general scheme for designing joint measurements, as shown in Ref. [13], where the Bell-like measurement is achieved by a new kind of probability operator-valued measure (POVM) that generalizes the joint measurements of position-momentum, or the measurement of the "direction" of the angular momentum.

We need to introduce some preliminary mathematics. Consider the heterodyne-current operator [14] $Z = a+b^\dagger$, which satisfies the commutation relation $[Z, Z^\dagger] = 0$ and the eigenvalue equation $Z|z\rangle\rangle_{ab} = z|z\rangle\rangle_{ab}$, with $z \in \mathbb{C}$. The eigenstates $|z\rangle\rangle_{ab}$ are given by [15,16]

$$|z\rangle\rangle_{ab} \equiv D_a(z)|0\rangle\rangle_{ab} = D_b(z^*)|0\rangle\rangle_{ab} \;, \tag{1}$$

where $D_d(z) = e^{zd^\dagger - z^* d}$ denotes the displacement operator for mode d and $|0\rangle\rangle_{ab} \equiv \frac{1}{\sqrt{\pi}}\sum_{n=0}^{\infty}(-1)^n|n\rangle_a \otimes |n\rangle_b$ on the Fock basis. The eigenstates $|z\rangle\rangle_{ab}$ are a complete orthogonal set with Dirac-delta normalization ${}_{ab}\langle\langle z|z'\rangle\rangle_{ab} = \delta^{(2)}(z-z')$. For $z = 0$ the state $|0\rangle\rangle_{ab}$ can be approximated by a physical (normalizable) state—so-called twin beam—corresponding to the output of a non-degenerate optical parametric amplifier (NOPA) in the limit of infinite gain [15].

For the following, it is also useful to evaluate the expression ${}_{cb}\langle\langle z|z'\rangle\rangle_{ab}$ which is given by

$$\begin{aligned}{}_{cb}\langle\langle z|z'\rangle\rangle_{ab} &= {}_{cb}\langle\langle 0|D_c^\dagger(z)D_a(z')|0\rangle\rangle_{ab} \\ &= \frac{1}{\pi}D_a(z')\mathcal{T}_{ac}D_c^\dagger(z) \;,\end{aligned} \tag{2}$$

where $\mathcal{T}_{ac} = \sum_n |n\rangle_a\, {}_c\langle n|$ denotes the *transfer* operator [13] satisfying the relation $\mathcal{T}_{ac}|\psi\rangle_c = |\psi\rangle_a$ for any vector $|\psi\rangle$. Here we briefly transpose the main results of the continuous variable cloning in Ref. [5], in a compact formalism suited to the following treatment. The input state at the cloning machine can be written

$$|\phi\rangle = |\varphi\rangle_c \otimes \int_{\mathbb{C}} d^2 z\, f(z,z^*) |z\rangle\rangle_{ab} \ , \tag{3}$$

where $|\varphi\rangle_c$ is the original in the Hilbert space $\mathcal{H}_c$, to be cloned in $\mathcal{H}_c$ itself and $\mathcal{H}_a$, whereas $\mathcal{H}_b$ is an ancillary Hilbert space. We do not specify for the moment the explicit form of the function $f(z,z^*)$. The cloning transformation is realized by the unitary operator [5]

$$U = \exp\left[(X_c + iY_c)\, Z^\dagger - (X_c - iY_c)\, Z \right] \ , \tag{4}$$

with X_c and Y_c denoting the conjugated quadratures for mode c, namely $X_c = (c + c^\dagger)/2$ and $Y_c = (c - c^\dagger)/2i$. Notice that one has $U|z\rangle\rangle_{ab} = D_c^\dagger(z)\,|z\rangle\rangle_{ab}$. The state after the cloning transformation is given by $|\phi\rangle_{out} = U|\phi\rangle$. Let us evaluate the one-site restricted density matrix ϱ_c and ϱ_a corresponding to the state $|\phi\rangle_{out}$, for the Hilbert spaces $\mathcal{H}_c$ and $\mathcal{H}_a$ supporting the clones. For ϱ_c one has

$$\begin{aligned}
\varrho_c &= \mathrm{Tr}_{ab}[|\phi_{out}\rangle\langle\phi_{out}|] \\
&= \int_{\mathbb{C}} d^2 w \int_{\mathbb{C}} d^2 z \int_{\mathbb{C}} d^2 z'\, f(z,z^*) f^*(z',z'^*) \\
&\times {}_{ab}\langle\langle w| D_c^\dagger(z) |\varphi\rangle_c\, {}_c\langle\varphi| D_c(z') \otimes |z\rangle\rangle_{ab}\, {}_{ab}\langle\langle z'|w\rangle\rangle_{ab} \\
&= \int_{\mathbb{C}} d^2 z |f(z,z^*)|^2 D_c^\dagger(z) |\varphi\rangle\langle\varphi| D_c(z) \ ,
\end{aligned} \tag{5}$$

where we have evaluated the trace by using completeness and orthogonality of the eigenstates $|w\rangle\rangle_{ab}$ of Z. For ϱ_a, using Eq. (2), one has

$$\begin{aligned}
\varrho_a &= \mathrm{Tr}_{cb}[|\phi_{out}\rangle\langle\phi_{out}|] \\
&= \int_{\mathbb{C}} d^2 w \int_{\mathbb{C}} \frac{d^2 z}{\pi} \int_{\mathbb{C}} \frac{d^2 z'}{\pi}\, f(z,z^*) f^*(z',z'^*) \\
&\times D_a(z) \mathcal{T}_{ac} \left[D_c^\dagger(w) D_c^\dagger(z) |\varphi\rangle_c\, {}_c\langle\varphi| D_c(z') D_c(w) \right] \\
&\times \mathcal{T}_{ca} D_a^\dagger(z') \\
&= \int_{\mathbb{C}} d^2 w\, |\stackrel{\sim}{f}(w,w^*)|^2 D_a^\dagger(w) |\varphi\rangle_a\, {}_a\langle\varphi| D_a(w) \ ,
\end{aligned} \tag{6}$$

where $\stackrel{\sim}{f}(w,w^*)$ denotes the Fourier transform over the complex plane $\stackrel{\sim}{f}(w,w^*) = \int_{\mathbb{C}} \frac{d^2 z}{\pi}\, e^{wz^* - w^* z}\, f(z,z^*)$. Hence, for $f(z,z^*) = \sqrt{2/\pi}\, e^{-|z|^2}$ one has two identical clones $\varrho_c = \varrho_a$, which are given by the original state $|\varphi\rangle$ degraded by Gaussian noise. The state preparation $|\chi\rangle$ pertaining to the Hilbert space $\mathcal{H}_a \otimes \mathcal{H}_b$ is given by [11]

$$\begin{aligned}
|\chi\rangle &= \sqrt{\frac{2}{\pi}} \int_{\mathbb{C}} d^2 z\, e^{-|z|^2} |z\rangle\rangle_{ab} = \frac{2\sqrt{2}}{3} \times \\
&\sum_{n=0}^{\infty} \left(-\frac{1}{3} \right)^n |n\rangle_a \otimes |n\rangle_b = e^{\mathrm{atanh}\frac{1}{3}(ab - a^\dagger b^\dagger)} |n\rangle_a \otimes |n\rangle_b \ .
\end{aligned} \tag{7}$$

One recognizes in Eq. (7), the twin-beam state at the output of a NOPA with total number of photons $N = \langle\chi| a^\dagger a + b^\dagger b |\chi\rangle = 1/4$.

Quantum cloning allows one to engineer new joint measurements. In fact, suitable measurements on the cloned copies are equivalent to a joint measurement on the original. Let us now consider a joint position-momentum on the original copy through the present scheme. More precisely, in our case measuring two quadratures on the two clones will be equivalent to the joint measurement of a couple of conjugated quadratures on the original, namely to a heterodyne measurement. This can be shown as follows. Consider the entangled state ϱ at the output of the cloning machine, after tracing over the ancillary mode b. One has

$$\begin{aligned}
\varrho &= \mathrm{Tr}_b[|\phi_{out}\rangle\langle\phi_{out}|] \\
&= \frac{1}{2} P_{ca} (|\varphi\rangle_c\, {}_c\langle\varphi| \otimes \mathbb{1}_a) P_{ca} \ ,
\end{aligned} \tag{8}$$

where P_{ca} is the projector given by

$$P_{ca} = V(|0\rangle_c\, {}_c\langle 0| \otimes \mathbb{1}_a) V^\dagger \ , \tag{9}$$

with $V = \exp[\frac{\pi}{4}(c^\dagger a - c a^\dagger)]$. Measuring the quadratures X_c and Y_a is then equivalent to perform the measurement on the original state $|\varphi\rangle_c$ described by the POVM

$$F(x,y) = \mathrm{Tr}_a[P_{ca} |x\rangle_c\, {}_c\langle x| \otimes |y\rangle_a\, {}_a\langle y| P_{ca}] \ , \tag{10}$$

where $|x\rangle_c$ and $|y\rangle_a$ denote the eigenstates of X_c and Y_a, respectively. From the following relations [16]

$$\begin{aligned}
& V^\dagger |x\rangle_c\, {}_c\langle x| \otimes |y\rangle_a\, {}_a\langle y| V \\
&= 2 |\sqrt{2}(x - iy)\rangle\rangle_{ca}\, {}_{ca}\langle\langle \sqrt{2}(x - iy)| \ , \\
& V|\alpha\rangle_c \otimes |\beta\rangle_a = |(\alpha + \beta)/\sqrt{2}\rangle_c \otimes |(\beta - \alpha)/\sqrt{2}\rangle_a \ , \\
& {}_c\langle 0|z\rangle\rangle_{ca} = \frac{1}{\sqrt{\pi}} |z^*\rangle_a \ ,
\end{aligned} \tag{11}$$

one obtains

$$F(x,y) = \frac{1}{\pi} |x + iy\rangle_c\, {}_c\langle x + iy| \ , \tag{12}$$

namely the coherent-state POVM, which is the well-known optimal joint measurement of conjugated quadratures X_c and Y_c [In Eq. (12) and in the last two lines of Eq. (11) single-mode vectors are coherent states].
Eqs. (8) and (9) allows one to show that the cloning machine here considered is covariant with respect to the Weyl-Heisenberg group, represented by the displacement operator. One has

$$\begin{aligned}
& \frac{1}{2} P_{ca} (D_c(\alpha) |\varphi\rangle_c\, {}_c\langle\varphi| D_c^\dagger(\alpha) \otimes \mathbb{1}_a) P_{ca} \\
&= D_c(\alpha) \otimes D_a(\alpha)\, \varrho\, D_c^\dagger(\alpha) \otimes D_a^\dagger(\alpha) \ .
\end{aligned} \tag{13}$$

In the following we show that the unitary evolution in Eq. (4) can be obtained from a network of three NOPA's under suitable gain conditions. We rewrite Eq. (4) as $U = \exp(B + A)$, with $B = ca^\dagger - c^\dagger a$ and $A = bc - b^\dagger c^\dagger$. Upon defining $C = ab - a^\dagger b^\dagger$, one easily checks the commutation relations $[C, A] = B$, $[C, B] = A$, and $[B, A] = C$. Hence, the following identity holds

$$e^{\lambda C}\, A\, e^{-\lambda C} = \cosh(\lambda) A + \sinh(\lambda) B \; . \tag{14}$$

From Eq. (14) one obtains the realization for the operator U

$$U = \lim_{\lambda\to\infty} e^{\lambda C}\, e^{2e^{-\lambda} A}\, e^{-\lambda C} \; . \tag{15}$$

Each term in the product of the r.h.s of Eq. (15) is realized by a NOPA. The continuous variable cloning from one to two copies is then achievable in the limit $\lambda \to \infty$ through the network of parametric amplifiers depicted in Fig. 1. Notice that the evolution operator for the generation of the input state of Eq. (7) can be absorbed into the last factor of the product in Eq. (15), yielding the overall unitary transformation

$$U' = e^{\lambda C}\, e^{2e^{-\lambda} A}\, e^{(\mathrm{atanh}\frac{1}{3} - \lambda) C} \; . \tag{16}$$

The gain values of the three amplifiers are constrained as follows

$$\begin{aligned} G_1 &= \cosh^2(\lambda - \mathrm{atanh}\; 1/3) \; , \\ G_2 &= \cosh^2(2e^{-\lambda}) \; , \\ G_3 &= \cosh^2 \lambda \; . \end{aligned} \tag{17}$$

Notice that a cascade of N networks could produce 2^N clones. However, as shown in Ref. [4], this is not an efficient way to produce multiple clones.

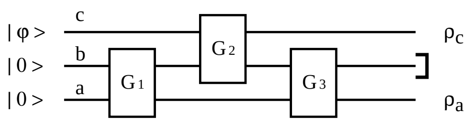

FIG. 1. Network of parametric amplifiers, each of them working as an input-output four-port gate, to realize the continuous variable one-to-two cloning. The values of the gain parameters are given in Eqs. (17).

In the following we provide the POVM corresponding to the measurement of two quadratures $X_c(\varphi) = (c^\dagger e^{i\varphi} + c e^{-i\varphi})$ and $X_a(\theta) = (a^\dagger e^{i\theta} + a e^{-i\theta})$ over the two clones, in the case of realistic cloning ($\lambda < \infty$). For $0 < \theta - \varphi < \pi$, one obtains the POVM

$$\begin{aligned} F_\lambda(x, x'; \varphi, \theta) = {} & \frac{1}{4\pi} \frac{C|\delta|^2}{\sqrt{CD - E^2}} \\ & \times S^\dagger(\xi)\, D(\alpha\,\delta)\, \varrho\, D^\dagger(\alpha\,\delta) S(\xi) \end{aligned} \tag{18}$$

where

$$\begin{aligned} \alpha &= -\frac{i}{2} x + \frac{C x' - E x}{2\sqrt{CD - E^2}} \; , \\ \varrho &= \frac{4}{C|\delta|^2 + 2} \left(\frac{C|\delta|^2 - 2}{C|\delta|^2 + 2} \right)^{c^\dagger c} \; , \\ C &= (\sinh\varepsilon\, \sinh\lambda')^2 + \frac{1}{2} \sinh^2\varepsilon \\ D &= (\cosh\lambda\, \cosh\lambda' - \sinh\lambda\, \cosh\varepsilon\, \sinh\lambda')^2 \\ &\quad + \frac{1}{2} (\sinh\lambda\, \sinh\varepsilon)^2 - \frac{1}{2} \\ E &= \cos(\varphi - \theta) [\sinh\varepsilon\, \sinh\lambda' (\cosh\lambda\, \cosh\lambda' \\ &\quad - \sinh\lambda\, \cosh\varepsilon\, \sinh\lambda') - \frac{1}{2} \cosh\varepsilon\, \sinh\lambda\, \sinh\varepsilon] \; , \\ \delta &= -(|\gamma|^2 - |\beta|^2)^{-1/2}\, e^{-i \arg\gamma} \\ \beta &= \frac{1}{4} \cosh\varepsilon\, e^{i\varphi} \left(i + \frac{E}{\sqrt{CD - E^2}} \right) \\ &\quad + \frac{1}{4} \sinh\lambda\, \sinh\varepsilon\, e^{i\theta} \frac{C}{\sqrt{CD - E^2}} \; , \\ \gamma &= \frac{1}{4} \cosh\varepsilon\, e^{i\varphi} \left(-i + \frac{E}{\sqrt{CD - E^2}} \right) \\ &\quad + \frac{1}{4} \sinh\lambda\, \sinh\varepsilon\, e^{i\theta} \frac{C}{\sqrt{CD - E^2}} \; , \\ \xi &= \mathrm{acosh}(|\gamma\delta|)\, e^{i(\arg\gamma + \arg\beta)} \end{aligned} \tag{19}$$

with $\varepsilon = -2e^{-\lambda}$ and $\lambda' = \lambda - \mathrm{atanh}\frac{1}{3}$.

Notice that for $\lambda \to \infty$ and $\theta - \varphi = \pi/2$ one gets the result in Eq. (12), namely one achieves the ideal POVM for simultaneous measurement of conjugated quadratures.

Now we can evaluate the added noise for simultaneous measurement of conjugated quadratures over the two clones. One has the following input-output relations between the expectation values over the two clones $\langle \cdots \rangle_o$ at the output and the same expectation $\langle \cdots \rangle_i$ for the original copy at the input

$$\begin{aligned} \langle X_c \rangle_o &= \cosh\varepsilon\, \langle X_c \rangle_i \; , \\ \langle Y_a \rangle_o &= \sinh\lambda \sinh\varepsilon\, \langle Y_c \rangle_i \; , \\ \langle X_c^2 \rangle_o &= \cosh^2\varepsilon\, \langle X_c^2 \rangle_i \\ &\quad + \frac{1}{4} \sinh^2\varepsilon (2\sinh^2\lambda' + 1) \; , \\ \langle Y_a^2 \rangle_o &= \sinh^2\lambda \sinh^2\varepsilon\, \langle Y_c^2 \rangle_i \\ &\quad + \frac{1}{4} (\cosh\lambda \cosh\lambda' - \sinh\lambda \sinh\lambda' \cosh\varepsilon)^2 \\ &\quad + \frac{1}{4} (\sinh\lambda \cosh\lambda' \cosh\varepsilon - \cosh\lambda \sinh\lambda')^2 \; . \end{aligned} \tag{20}$$

In the limit of $\lambda \to \infty$ one has

$$\begin{aligned} \langle \Delta X_c^2 \rangle_o &\to \langle \Delta X_c^2 \rangle_i + \frac{1}{4} \; , \\ \langle \Delta Y_a^2 \rangle_o &\to \langle \Delta Y_c^2 \rangle_i + \frac{1}{4} \; , \end{aligned} \tag{21}$$

which proves the optimality of the joint measurement [17].

The behavior of the product of variances for the simultaneous measurement of X_c and Y_c via homodyne detection over clones is plotted in Fig. 2, for arbitrary coherent state (for which $\langle \Delta X_c^2 \rangle_i = \langle \Delta Y_c^2 \rangle_i = 1/4$). Notice that for increasing value of λ the optimality of the joint measurement is rapidly achieved.

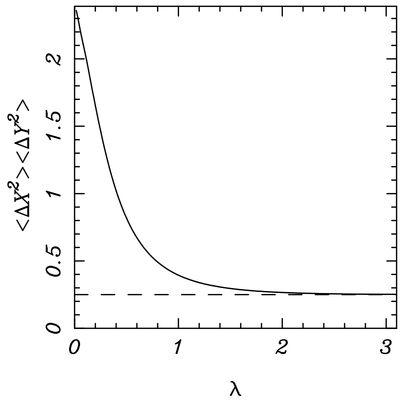

FIG. 2. Optimal joint measurement of conjugated quadratures via continuous variable cloning and homodyne detection, for arbitrary coherent input state. The bound $\langle \Delta X^2 \rangle \langle \Delta Y^2 \rangle = 1/4$ is achieved by increasing the parameter λ which provides the optimal cloning for $\lambda \to \infty$ (the plot is independent of the amplitude of the coherent state).

The condition $f(z,z^*) = \widetilde{f}\,(z,z^*)$ for Eqs. (5) and (6) in order to obtain identical clones can be satisfied also by a bivariate Gaussian of the form

$$f(z,z^*) = \sqrt{\frac{2}{\pi}}\,\exp\left(-\frac{\mathrm{Re}^2 z}{\sigma^2} - \sigma^2\,\mathrm{Im}^2 z\right) . \qquad (22)$$

In such case, as shown in Ref. [11], the cloning transformation becomes optimal for the joint measurement of noncommuting quadratures X_ϕ and $X_{-\phi}$, at angles which depend on the parameter σ in Eq. (22) as $\phi = \mathrm{arctg}(\sigma^2)$.

As regards the experimental realization of the network in Fig. 1, the scheme which is presently engineered in our lab in Rome works with an injection method similar to the one used in the implementation of the all-optical Schroedinger-cat of Ref. [8]. Three identical equally oriented nonlinear crystals of beta-barium-borate cut for Type II phase matching are excited by coherent beams derived from a common UV beam at wavelength $\lambda_p = 400nm$. In the present experiment the UV beam is supplied by second harmonic generation of the output of a Coherent MIRA TI:SA mode-locked laser consisting of a train of 150 fs pulses emitted at a rate of 76 MHz. The average output power does not exceed 0.6 W, and then the amplification gain is of the order $g \simeq 0.02$, where $G = \cosh^2 g$ in Eqs. (17). We expect a far larger efficiency by the forthcoming implementation within the apparatus of a regenerative NOPA Coherent REGA9000. In this case the value of g is multiplied by an adjustable factor in the range $10-50$, and the cloning efficiency is expected to increase by the same factor. The nonlinear crystals emit π-entangled photons with wavelength $\lambda = 800nm$ over two modes determined by two fixed $1mm$ pinholes placed 2 meters away from the source crystals. The conditions imposed by Eqs. (17) are achieved by a precise setting of the intensity of the three single-mode UV pump beams by use of adjustable Circular Neutral Density Filters Newport 946. Great care is taken in space-mode filtering which select the injection modes through the pinholes. The spatio-temporal superposition for such short pulses and mode matching at homodyne detectors are the main experimental challenges.

In conclusion, our experimental scheme based on a network of three NOPA's is designed for engineering quantum clones of harmonic oscillator states. For increasing gain, optimality is achieved in performing joint quadrature measurements via cloning.

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